Fall 1996, Physics 222: Homework set #7 (Jian Wang)

Ch20, P34

An emf of 24 mV is induced in a 500-turn coil at an instant when the current is 4 A and is changing at the rate of 10 A/s. What is the magnetic flux through each turn of the coil?

Solution:

The inductance of a coil containing N turns is $L = \frac{N\Phi_m}{I}$.

Also: $$L = -\frac{e}{\frac{dI}{dt}}$$

Therefore:

$$\frac{N\Phi_m}{I} = -\frac{e}{\frac{dI}{dt}}$$

Using magnitude:

$$\Phi_m = \frac{Ie}{N\frac{dI}{dt}}$$

where Φ_m is the magnetic flux through each turn of the coil.

$$\Phi_m = \frac{4A \times 24 \times 10^{-3}\ \mathrm{V}}{500 \times 10\ \mathrm{A/s}} = 1.92 \times 10^{-5}\ \mathrm{Tm}^2$$

Ch20, P37

A current, $I = I_0 \sin wt$, with I_0=5 A and $w/2p$ =60 Hz, flows through an inductor whose inductance is 10 mH. What is the back emf as a function of time?

Solution:

$$I = I_0 \sin wt = (5A)\sin(60 \times 2pt) = (5A)\sin(377t)$$

The back emf is

$$e_L = -L\frac{dI}{dt} = -10 \times 10^{-3} H \times \frac{d}{dt}\left[(5A)\sin 377t\right]$$

$$= -10 \times 10^{-3} H \times 5A\frac{d}{dt}(\sin 377t)$$

$$= (-18.8V)\cos 377t$$

Ch20, P44

Consider the circuit shown in Figure 20.40, taking $\mathcal{E}$=6 V, L=8 mH, and R=14 Ω. (a) What is the inductive time constant of the circuit? (b) Calculate the current in the circuit 250 μs after the switch is closed. (c) What is the value of the final steady-state current? (d) How long does it take the current to 80% of its maximum value?

Solution:

(a) Time constant of RL circuit is

$$t = \frac{L}{R} = \frac{8\times10^{-3}\text{ H}}{4\ \Omega} = 2\times10^{-3} s = 2\text{ ms.}$$

(b) The current in a RL circuit is

$$I(t) = \frac{e}{R}(1-e^{-t/t}) = \frac{6\text{ V}}{4\ \Omega}(1-e^{-250\times10^{-6}\text{ s}/2\times10^{-3}\text{ s}}) \approx 0.176\text{ A.}$$

(c) The final steady-state current is the value of the current when $t \to \infty$. Therefore,

$$I_f = \frac{e}{R} = \frac{6\text{ V}}{4\ \Omega} = 1.5\text{ A}$$

(d) $I_{\max} = I_f = 1.5\text{ A}$

$$I(t) = I_{\max}(1-e^{-t/t})$$

$$\Rightarrow \quad t = -t\ \ln[1-\frac{I(t)}{I_{\max}}] = -2\times10^{-3} s \ln[1-80\%] \approx 3.22\times10^{-3}\text{ s} = 3.22\text{ ms}$$

Ch20, P48

One application of an RL circuit is the generation of high-voltage transients from a low-voltage dc source, as shown in Figure 20.41. (a) What is the current in the circuit a long time after the switch has been in position A? (b) Now the switch is thrown quickly from A to B. Compute the initial voltage across each resistor and the inductor. (c) How much time elapses before the voltage across the inductor drops to 12 V?

Solution:

(a) When the switch is in position A, the circuit is

Figure here.

$$I_f = \frac{e}{R} = \frac{12V}{12\Omega} = 1A$$

(b) When the switch is thrown to B, the circuit becomes

We use the following equation:

$$I(t) = \frac{e}{R} e^{-t/t}$$

where $\frac{e}{R}$ is the initial current in the circuit.

The initial current in circuit (b) is the final current in circuit (a). So

$$I_i(b) = I_f(a) = 1A$$

$$V_{1200\Omega} = IR = 1A \times 1200\Omega = 1200V = 1.2kV$$

$$V_{12\Omega} = IR = 1A \times 12\Omega = 12V$$

$$e_L + IR = 0 \;\Rightarrow\; e_L = V_{1200\Omega} + V_{12\Omega} = 1212V = 1.212kV$$

(c) $e_L(t) = I(t)R = R \times \frac{e}{R} e^{-t/t} = e_L e^{-t/t}$ (use magnitude only)

$$\Rightarrow\; t = -t \ln[\frac{e_L(t)}{e_L}] = -\frac{2H}{1200\Omega + 12\Omega} \ln(\frac{12V}{1212V}) \approx 7.62ms$$

Ch20, P52

At t=0, a source of emf, $\mathcal{E}$=500 V, is applied to a coil that has an inductance of 0.80 H and a resistance of 30 Ω. (a) Find the energy stored in the magnetic field when the current reaches half its maximum value. (b) How long after the the emf is connected does it take for the current to reach this value?

Solution:

(a) The energy stored in the magnetic field of an inductor carrying a current I is

$$U_m = \frac{1}{2} L I^2$$

In this case,

$$I_{\max} = \frac{e}{R} = \frac{500V}{30\Omega} = \frac{50}{3} A$$

$$I = \frac{1}{2} I_{\max} = \frac{25}{3} A$$

$$\Rightarrow U_m = \frac{1}{2} \times 0.8H \times (\frac{25}{3} A)^2 \approx 27.8J$$

(b) $I(t) = \frac{e}{R}(1 - e^{-t/t})$

$$\Rightarrow t = -t \ln[1 - \frac{I(t)}{I_{\max}}]$$

$$= -\frac{0.8H}{30\Omega} \times \ln(1 - 0.5) \approx 1.85 \times 10^{-2} s = 18.5ms$$

Ch20, P53

The magnetic field inside a superconducting solenoid is 4.5 T. The solenoid has an inner diameter of 6.2 cm and a length of 26 cm. (a) Determine the magnetic energy density in the field. (b) Determine the magnetic energy stored in the magnetic field within the solenoid.

Solution:

(a) The maganetic energy density is

$$u_m = \frac{B^2}{2m_0} = \frac{(4.5T)^2}{2 \times (4p \times 10^{-7} T \cdot m / A)} \approx 8.06 \times 10^6 J/m^3$$

(b) The magnetic energy stored in the field equals u_m times the volume of the solenoid (the volume in which B is non-zero).

$$U_M = u_m V = (8.06 \times 10^6 J/m^3) \times [p \times (\frac{6.2 \times 10^{-2}}{2} m)^2 \times 26 \times 10^{-2} m$$

$$\approx 6.32 \times 10^3 J = 6.32kJ$$